



THINKING SKILLS

Paper 3 Problem Analysis and Solution

May/June 2025

You must answer on the enclosed answer booklet.

You will need: Answer booklet (enclosed)
Calculator

- Answer **all** questions.
- Follow the instructions on the front cover of the answer booklet. If you need additional answer paper, ask the invigilator for a continuation booklet.
- You should use a calculator where appropriate.
- Show your working.
 - Where a final answer is incorrect or missing, you may still be awarded marks for correct steps towards a solution.
 - In some questions, if you do not show your working, full marks will not be awarded.

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages. Any blank pages are indicated.

- 1 *Your Choice* is a TV general knowledge quiz. Three contestants take part in each show.

The three contestants are asked the same 40 questions, each with four answer options, A, B, C and D to choose from. Each contestant has a keypad, linked to a central computer, with which to register their choices.

\$100 is awarded for every question that is answered correctly, distributed according to how many contestants give the correct answer.

- If only one contestant gives the correct answer, that contestant receives \$100.
- If two contestants give the correct answer, they both receive \$50.
- If all three contestants give the correct answer, the first to register their choice receives \$40 and the other two receive \$30 each.

After the 40th question, the contestant with the greatest amount of money is declared the winner of the show and progresses to the final stage, known as *The Multiplier*.

On a recent show, for the first time all three contestants answered all 40 questions correctly. Ian and Diane both finished with the same total amount of money as each other, but Trina was the winner by \$70.

- (a) What was Trina's winning total? [2]

In the show currently being recorded, the totals after the 40th question were:

| | |
|--------|--------|
| Una | \$1290 |
| Duane | \$1270 |
| Tracey | \$1240 |

Only two of the 40 questions were not answered correctly by **any** of the contestants.

- (b) Explain how this can be deduced. [2]

Una was in last place until she was the only contestant to answer question number 40 correctly and received the only \$100 award of the show.

Tracey was disappointed not to win. Her total of 34 correct answers was greater than either of the others, but she was not the first to register her answer for any of the questions that were answered correctly by all three contestants.

- (c) (i) How many questions were answered correctly by all three contestants? [3]
 (ii) How many questions were answered correctly by two of the three contestants? [1]

In *The Multiplier*, the contestant starts with their winning total and receives 10 further questions. \$50 is deducted from the total for every question that the contestant chooses not to give an answer to and \$100 is deducted for every incorrect answer. The contestant wins the final total multiplied by the number of 'multiplier' questions answered correctly.

Una is hoping to win at least \$5000. However, she has just answered the first 'multiplier' question incorrectly. She has now decided that she will only give an answer to a question if she knows it is correct.

- (d) Assuming that any answer that Una gives will be correct, what is the minimum number of questions she must answer in order to win at least \$5000? [2]

[Turn over for Question 2]

- 2 Grace is a children's entertainer who can be booked for parties. When she receives a request to perform at a party, Grace collects the following information:

- The number of hours for which she will need to perform.
- The distance that she will need to travel to reach the party.
- Her own rating for the party, to indicate how much she thinks she will enjoy performing. The rating is either 1, 2 or 3. A rating of 3 is given to the parties that she will most enjoy performing at.
- The fee that she will be paid for performing at the party, which must always be a whole number of dollars.

Grace uses the following method to calculate a score for each request:

- She subtracts the number of kilometres that she will need to travel to reach the party from the fee that she will be paid for performing.
- She then divides this value by the number of hours for which she will perform.
- If her rating for how much she would enjoy performing at the party is 1 then she reduces this amount by 10%.
- If her rating for how much she would enjoy performing at the party is 3 then she increases this amount by 10%.

Grace will **not** perform at any party with a score of less than 25. If she has more than one request scoring 25 or more for the same day then she will choose the one with the highest score.

Grace has four requests to perform at parties next Saturday. The details are shown in the table.

| <i>Customer</i> | <i>Length (hours)</i> | <i>Fee (\$)</i> | <i>Distance (km)</i> | <i>Grace's rating</i> |
|-----------------|-----------------------|-----------------|----------------------|-----------------------|
| Mollie | 3 | 88 | 10 | 1 |
| John | 2 | 68 | 6 | 2 |
| Frank | 3 | 99 | 12 | 3 |
| Wendy | 4 | 135 | 9 | |

Grace has not yet decided on her rating for Wendy's party.

- (a) Show that Grace will **not** perform at Mollie's party. [2]

- (b) Show that Grace will **not** perform at John's party. [2]

Once she had allocated a rating to Wendy's party, Grace used her system to decide that she would perform at Frank's party.

- (c) What rating or ratings might Grace have given to Wendy's party? [1]

When Grace contacted John to tell him that she would not perform at his party, John offered to increase the fee.

- (d) What is the smallest fee that John could offer so that Grace would choose to perform at his party? [2]

Grace decides that she would like to change her system so that she is more likely to choose longer parties. To do this she calculates the score as before, but then adds on a fixed amount for each hour that the party lasts.

To decide on this fixed amount, Grace considers parties that she would need to travel 5km to reach and that she would award a rating of 2. Initially, she wants to set the additional amount for each hour so that a 2-hour party with a fee of \$69 will receive the same score as a 4-hour party with a fee of \$109.

(e) What is the amount that Grace would need to set for each hour that the party lasts? [2]

Instead, Grace decides to set the fixed amount for each hour that the party lasts to 5. She realises that she needs to change the minimum score that a party needs to be given in order for her to choose to perform at it. She would like to choose a value that ensures that she will reject the same 3-hour parties as she would have rejected under her original system.

(f) What is the minimum value that a party will need to score for Grace to perform at it? [1]

(g) Show that, under the new system, Wendy's party would have been chosen no matter what rating Grace gave it. [2]

Grace considers the two parties shown below:

| <i>Customer</i> | <i>Length (hours)</i> | <i>Fee (\$)</i> | <i>Distance (km)</i> | <i>Grace's rating</i> |
|-----------------|-----------------------|-----------------|----------------------|-----------------------|
| Polly | 5 | 180 | 5 | 1 |
| Quentin | 4 | | 7 | 2 |

(h) What fee would need to be offered for Quentin's party in order for both parties to receive the same score under the new system? [3]

- 3 Roadworks have blocked the main road, and all drivers are following the instructions of one of two navigation systems to determine whether to go left or right at the junction before the blockage. One system, Beng, is alternately sending its users left (L) and right (R) to spread the traffic. The other, Dikdik, is repeatedly sending one of its users left and then the next two right. Neither system takes account of what the other one is doing.

- (a) If each driver is as likely to use one system as the other, what proportion turn left? [1]
- (b) What is the shortest list of consecutive turns that could **never** be seen? [1]
- (c) The first six drivers are sent L, L, R, L, R, R (in that order).

For two of these drivers, the system that they are using can be determined.

Identify these two drivers and which system each is using, and explain how it can be known. [3]

- (d) Give an example of a sequence of nine consecutive turns with the smallest possible number of left turns. Indicate which system is used by each driver. [1]
- (e) What is the smallest possible number of right turns in any nine consecutive turns? Give an example of this which has each system used by at least three drivers, indicating which system is used by each driver. [2]
- (f) If 57 out of 156 drivers turned left, estimate what proportion used Beng. [2]

[Turn over for Question 4]

- 4 A company offers a service to help workers in the local city to travel. There is a large car park outside the city and a bus travels between the car park and the city centre during the day. Tickets must be bought to travel from the car park to the city, but journeys back to the car park are free.

There is also a 3-day ticket available, which can be used for one journey to the city on each of three consecutive days. Similarly, a 5-day ticket is available, which can be used for one journey to the city on each of five consecutive days.

The service operates each day from Monday to Friday, but the 3-day and 5-day tickets include Saturdays and Sundays when working out the consecutive days.

The prices for the types of tickets are:

| <i>Ticket type</i> | 1-day | 3-day | 5-day |
|--------------------|-------|-------|-------|
| <i>Price (\$)</i> | 8 | 20 | 30 |

This week, Jack needs to travel to the city on Monday, Tuesday, Wednesday and Friday.

- (a) What is the minimum that Jack would have to pay for his journeys? [1]

This week, Jill needs to travel to the city on Monday, Tuesday, Thursday and Friday.

- (b) What is the minimum that Jill would have to pay for her journeys? [1]

The table below shows the number of journeys made to the city and the total amount of money paid for tickets on each day last week.

| | <i>Monday</i> | <i>Tuesday</i> | <i>Wednesday</i> | <i>Thursday</i> | <i>Friday</i> |
|-----------------------------|---------------|----------------|------------------|-----------------|---------------|
| <i>Journeys to the city</i> | 44 | 81 | 125 | 130 | 92 |
| <i>Total paid (\$)</i> | 1036 | 844 | 1068 | 112 | 136 |

All of the tickets that were used last week were bought last week.

Every ticket that was bought last week was used for a journey to the city on that day.

- (c) Based **only** on the information for Monday:

- (i) What is the **maximum** number of 5-day tickets that could have been sold on Monday? State also the number of 3-day and 1-day tickets that would have been sold in this case. [4]

- (ii) What is the **minimum** number of 5-day tickets that could have been sold on Monday? State also the number of 3-day and 1-day tickets that would have been sold in this case. [2]

It is known that customers always buy tickets such that they pay the least necessary for their journeys. Consequently, Monday was the only day last week on which 5-day tickets were bought.

The tickets that were used for journeys on Friday were:

1-day tickets bought on Friday,
or 3-day tickets bought on Wednesday,
or 5-day tickets bought on Monday.

(d) What are the tickets that might have been used for the journeys on Thursday? For each possibility, state the type of ticket and the day that it was bought. [1]

Janet realises that, from the information in the table, she can deduce exactly how many of each type of ticket were sold on each day of last week.

(e) (i) Show that three 1-day tickets were sold on Tuesday. [3]

(ii) How many 5-day tickets were sold on Monday? [3]

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